Erratum: Frustrated classical Heisenberg model in one dimension with nearest-neighbor biquadratic exchange: Exact solution for the ground-state phase diagram [Phys. Rev. B 80, 012407 (2009)]

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Three errors are reported here. The last two are inconsequential. The first is consequential.

(i) As the title says, the ground state phase diagram for classical spins on a linear chain with nearest-neighbor (nn) and antiferromagnetic (AF) next-nearest-neighbor (nnn) Heisenberg interactions (producing a frustrated Heisenberg model), plus nn biquadratic terms, was determined exactly (in the thermodynamic limit). A remark was made concerning what happens if the nnn interaction becomes ferromagnetic; the statement was erroneous. Here the exact phase diagram is extended to include ferromagnetic nnn interactions, thereby correcting the error.

The list of cluster energies, Eq. (7) in the original paper, to be denoted as [K] for the various stationary solutions, needs one additional one, significant for ferromagnetic nnn interactions ($\gamma < 0$). The completed list is

$$h_{ferro} = h_c(0,0) = -1 - a + \gamma$$
$$h_{uudd} = h_c(0,\pi) = -a - \gamma$$
$$h_{spiral} = h_c(\theta_0, -\theta_0) = -\gamma - \frac{1}{4(2\gamma - a)}$$
$$h_{cantedferro} = h_c(\overline{\theta}_0, \overline{\theta}_0) = \frac{1}{4a} + \gamma,$$

where $\cos \theta_0 = 1/(2(2\gamma - a))$, $\cos \overline{\theta}_0 = -1/(2a)$. The last two energies are valid only for real θ_0 , $\overline{\theta}_0$. The cluster state $(\theta, \theta') = (\overline{\theta}_0, \overline{\theta}_0)$ propagates as the state where, e.g., the even site spins are "up," the odd site spins are canted by the angle $\overline{\theta}_0$ (a two-sublattice state). [A cluster state $(\theta, -\theta)$ propagates as a spiral with turn angle θ .] Comparison of these energies leads to Fig. 1 below.

As in [K], these solutions were tested against the possibility of having missed a stationary solution by calculating the cluster energy $h_c(\theta, \theta')$ over a mesh of values of the angles, each going over $-\pi, \pi$, at a sample of points in each region of the phase diagram. This procedure was now followed for the region at $\gamma < 0$, in addition to the other three regions.

(ii) Equation (6) of [K] has a misprint; it should read

$$\cos \theta_0 = \frac{1}{2(2\gamma - a)} \quad \text{for } |2(2\gamma - a)| \ge 1.$$

$$\tag{1}$$

(iii) The statement, "in that work only a < 0, $\gamma = 0$ is considered," referring to Ref. 17 (M. F. Thorpe and M. Blume) of [K], is incorrect, but is irrelevant for the problem considered.



FIG. 1. (Color online) Phase diagram: a vs γ . Disorder occurs on the emphasized vertical line segments.